LOGIC DEFINITIONS: WEEK 1

1. PROPOSITIONS

A ***proposition*** is a claim of fact. It is true or false, depending on whether the facts are indeed as it claims. Philosophers disagree about what propositions are. I say they are *messages*, mental items encoded into sentences and conveyed from mind to mind in conversation. Halbach says they are the *sentences* themselves.

2. ARGUMENTS, PREMISES, AND CONCLUSIONS

An ***argument*** is a purported deduction of a single proposition (called the ***conclusion***) from a set of propositions (called the ***premises***). Every argument—in our technical sense—must have exactly one conclusion. For convenience, we allow there to be any number of premises, even zero. (If you find the idea of an argument with no premises weird, then good for you: it is indeed weird. But don't worry about it—it's just one of those funny conventions logicians adopt for the sake of overall neatness.)

Halbach's definition is as follows:

> DEFINITION 1.8. *An argument consists of a set of declarative sentences (the premises) and a declarative sentence (the conclusion) marked as the concluded sentence.* (p. 17)

I don't like this definition. For one thing, the premises and conclusion of an argument are *propositions*, and propositions (I maintain) are the messages encoded into sentences, not the sentences themselves. Furthermore, in an argument, it is not just that the conclusion is *marked* as such; it is actually *concluded*, i.e. *purportedly deduced* from the premises. But I didn't write the syllabus. It is Halbach's definition that you must learn and be prepared to reproduce in the exam.

3. INTUITIVE VALIDITY

This notion isn't explicit in Halbach's book, but I want you to bear it in mind, and—crucially—*never to confuse it with formal or logical validity* (see number 6 below). An argument is ***intuitively valid*** if and only if the conclusion *follows* from the premises, or the premises *make* the conclusion true, or something of that nature (there are a few metaphors we generally use in describing this property).

4. LOGICAL FORM

The ***logical form*** of a proposition or argument is what you get when you abstract away what Halbach calls the "subject-specific" details. For example, consider the following two arguments:

> *All frogs hate blenders.*
> *Anything that hates blenders is technophobic.*
> *Therefore, all frogs are technophobic.*

All logicians are emotionally stunted.
All emotionally stunted people swoon to Coldplay.
Therefore, all logicians swoon to Coldplay.

It should be apparent to you that these arguments have the same overall structure, which we may represent schematically as follows:

All As are Bs.
All Bs are Cs.
Therefore, All As are Cs.

This is the ***logical form*** of these arguments. It should also be apparent to you that any argument of this form is intuitively valid. It is also *formally* valid, come to that; but we won't be coming to that until we have defined one more thing.

5. INTERPRETATIONS OF A PROPOSITION OR ARGUMENT

Halbach frequently talks about the ***interpretations*** of a proposition or argument.[1] This strikes me as a rather unfortunate choice of terminology, which you may find somewhat confusing—though the notion itself shouldn't be difficult. An ***interpretation*** of a proposition or argument is just any proposition or argument of the same logical form. Note that this includes the very proposition or argument in question, since every proposition or argument (trivially) has the same logical form as itself.

I find it helps to think of Halbach's interpretations as being interpretations of the *logical form* of a proposition or argument, rather than of that proposition or argument itself. So whenever Halbach says something about "an interpretation of this premise", for example, think to yourself: "an interpretation *of the form* of this premise". This terminology is perhaps a little more natural. Thus the form of a premise might be something like: *all As are Bs*. And then the interpretations of this form will be all the concrete instances of the abstract schema: *all politicians are liars*, *all bankers taste delicious with onions and gravy*, *all PPEists are brilliant at logic*, and so on.

6. FORMAL OR LOGICAL VALIDITY

An argument is ***formally* or *logically valid*** (we use these terms interchangeably) if and only if there is no argument of the same logical form that has true premises and a false conclusion. Or, in Halbach-speak:

CHARACTERISATION 1.9 (LOGICAL VALIDITY). *An argument is logically valid if and only if there is no interpretation under which the premises are all true and the conclusion is false.* (p. 19)

[1] Here and throughout, I will say "proposition" where Halbach says "sentence". This shouldn't bother Halbach, since he takes it that propositions *are* sentences. The contrary practice does bother me, however, since I insist that propositions are *not* sentences, but the messages encoded into sentences, as already noted.

It is *absolutely vital* that you get a solid grasp of this notion, and learn how to spot when an argument is or is not formally valid. If you master just one thing this week, let it be this. For the whole course is centred on the idea of formal validity, and any confusions you have about it will lead to knock-on confusions elsewhere. Bear in mind—a frequent difficulty for those new to this game—that *not every intuitively valid argument is formally valid*, and *not every formally valid argument is intuitively valid*. When assessing the formal validity of an argument, you need for the most part to *forget* your instinctive feelings about its intuitive validity, and just focus very precisely on the definition. Ask yourself: is there an argument of the same logical form that has true premises and a false conclusion?

In a formally valid argument, we say that the premises ***logically entail*** the conclusion. This corresponds to our intuitive notion of the conclusion *following from* the premises; but again, be careful always to keep the intuitive notion and the formal notion distinct in your minds.

7. FORMAL OR LOGICAL CONSISTENCY AND INCONSISTENCY

Now comes the first of a handful of notions closely related to that of formal validity. In each case there is a corresponding intuitive notion as well (though I will not bother defining these explicitly here). As in the case of validity, you must not confuse the formal notion with the intuitive notion, for the two never line up exactly.

First of all, a set of propositions is ***formally or logically consistent*** (again, we use these terms interchangeably) if and only if there is at least one set of the same logical form in which all of the propositions are true. This could of course be the very set we started with, though it need not be. In Volkerese:

> CHARACTERISATION 1.10 (CONSISTENCY). *A set of sentences is logically consistent if and only if there is at least one interpretation under which all sentences of the set are true.* (p. 23)

A set of propositions is ***formally or logically inconsistent***, meanwhile, if and only if it isn't consistent, i.e. if and only if there is *no* set of the same logical form in which all of the propositions are true (or equivalently: in every set of the same logical form, at least one of the propositions is false).

8. THE COUNTEREXAMPLE SET

The ***counterexample set*** of an argument is the set consisting of the premises together with the negation (i.e. the *denial* or *rejection*) of the conclusion. Halbach mentions the counterexample set of an argument in passing (p. 23), but doesn't draw attention to it with an overt label or definition. It is a very important notion, however, and deserves a label.

Why is this set so important? Think it through: If the argument is formally valid, then there will be no argument of the same form with true premises and a false conclusion. But now, if the conclusion is false, then the negation of the conclusion is true. This means that, if the argument is valid, there will be no argument of the same form in which the premises and the negation of the conclusion are all true. In other words, if the argument is valid, then its counterexample set must be inconsistent. And *vice versa* too: if the counterexample set is inconsistent, then the argument must be valid (just apply the same reasoning in reverse).

9. FORMAL OR LOGICAL TRUTH AND FALSEHOOD

A proposition is a ***formal or logical truth*** if and only if every proposition of the same logical form is true. A logical truth is also called a ***tautology*** (just another word for exactly the same thing). Logical truths are all true, of course, since every proposition is trivially of the same logical form as itself. In Halbachean:

> CHARACTERISATION 1.11 (LOGICAL TRUTH). *A sentence is logically true if and only if it is true under any interpretation.* (p. 24)

Contrariwise, a proposition is a ***formal or logical falsehood*** if and only if every proposition of the same logical form is false. A logical falsehood is also called a ***contradiction***. Thus Halbach:

> CHARACTERISATION 1.12 (CONTRADICTION). *A sentence is a contradiction if and only if it is false under any interpretation.* (p. 24)

A proposition that is neither a logical truth nor a logical falsehood is called a ***(formal or logical) contingency***. For some reason, this isn't mentioned in Halbach's book. To be clear, a proposition is a ***contingency*** if and only if there is at least one true proposition of the same form, and one false proposition of the same form.

10. FORMAL OR LOGICAL EQUIVALENCE

Finally, two propositions are said to be ***formally or logically equivalent*** if and only if they have the same truth value (i.e. either they are both true, or they are both false) no matter how you flesh out their logical form. In Volkerish:

> CHARACTERISATION 1.13 (LOGICAL EQUIVALENCE). *Sentences are logically equivalent if and only if they are true under exactly the same interpretations.* (p. 24)

If two propositions are logically equivalent, then the first logically entails the second, and the second logically entails the first.

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And there it is, and there you are, and there you have it. In addition to your exercises this week, you should also be doing your best to ensure that you have grasped all of these notions. Don't worry if they don't sink in all at once—it takes some practice to get the hang of them, and you'll be getting plenty of practice as term proceeds. Be sure to bring any questions with you to the class if something in particular isn't making sense.